- CLEAR AND CONCISE
- GRADED EXERCISES
- COMPREHENSIVE
- VERY USER FRIENDLY
- SOLUTIONS AT BACK

ADVANCED EDITION

UNDERSTANDING YEAR 9 MATHS

SAMPLE

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NOTE: The Australian National Curriculum has been split into 3 major strands:

1. Number & Algebra	2. Measurement & Geometry	3. Statistics & Probability
(NA)	(MG)	(SP)

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FINANCIAL MATHEMATICS

The 'Australian Curriculum Mathematics' (ACM) references for this sub-strand of 'Number and Algebra' (NA) are given below. This chapter may contain additional extension work which the author feels will be beneficial to the student.

- *Terminating and recurring decimals (ACMNA 184).*
- *Ratios and rates (ACMNA 188).*
- *Percentages and money (ACMNA 189).*
- *Converting rates (ACMNA 208).*
- *Simple interest (ACMNA 211).*
- *Wages, salaries, overtime rates, income tax (NSW).*

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'The most certain way to succeed is to just try one more time.'

- Thomas Edison (famous inventor)

SAMPLE

TERMINATING AND RECURRING DECIMALS

The decimals shown on the right are called **TERMINATING DECIMALS** because they all have a definite end point.

$$\frac{3}{4} = 0.75$$

$$\frac{12}{20} = 0.6$$

Example 1: Change $\frac{3}{8}$ to a decimal.

Write 3 as a decimal = 3.000

$\frac{3}{8}$ is the same as 3.000 ÷ 8

$$8\overline{)3.^{3}0\,^{6}0\,^{4}0} \quad 0.\ 3\ \ 7\ \ 5$$

Therefore $\frac{3}{8} = 0.375$



However, when we do the division shown above, we sometimes get decimal answers that never end.

Example 2: Write $\frac{1}{3}$ as a decimal.

$$3\overline{)1.^{1}0\,^{1}0\,^{1}0\,^{1}0} \quad 0.\ 3\ \ 3\ \ 3\ \ 3.....$$

$$\therefore \frac{1}{3} = 0.\dot{3}$$



We place a dot above the 3 to show that this number keeps on repeating infinitely.

When a decimal keeps repeating, it is called a REPEATING or RECURRING DECIMAL.
This is written with a dot on the repeating numbers.

Example 3: Change $\frac{5}{11}$ to a decimal.

Write 5 as a decimal = 5.000

$$11\overline{)5.^{5}0\,^{6}0\,^{5}0\,^{6}0\,^{5}0\,^{6}0\,^{5}0} \quad 0.\ 4\ \ 5\ \ 4\ \ 5\ \ 4\ \ 5....$$

$$\Rightarrow \frac{5}{11} = 0.\dot{4}\dot{5}$$

We place a dot on both the 4 and 5 to show that the pattern 45 repeats itself infinitely.

SAMPLE

Example 4: Change $\frac{254}{999}$ to a decimal.

Try doing this on your calculator and you should obtain the following answer:

254 ÷ 999 = 0.254254254254.....



To show that 254 is a repeating pattern we place a dot over the 2 and also over the 4.

$\therefore$ 0.254 254 254.... = $0.\dot{2}5\dot{4}$

Sometimes you will be required to convert a recurring decimal back into a fraction.

This is done using a clever method involving equations. It is best understood by following the examples below.

Example 5: Convert the following recurring decimals to fractions:

a) $0.\dot{5}$ b) $0.\dot{3}\dot{2}$

Solution: a) Let $x = 0.\dot{5} = 0.55555.....$

$\therefore$ $10x = 5.55555.....$

$10x - x = 5.555.... - 0.5555....$

$\therefore$ $9x = 5$

$\therefore$ $x = \frac{5}{9}$

Subtract the first equation from the second in order to cancel the recurring 5's.



This answer can easily be checked by using your calculator and dividing 5 by 9.

b) Let $x = 0.323232.....$

$\therefore$ $100x = 32.323232.....$

Subtract the first line from the second line

$\therefore$ $100x - x = 32.3232.... - 0.3232.....$

$\therefore$ $99x = 32$

$\therefore$ $x = \frac{32}{99}$

We multiply by 100 because 2 digits are recurring.



Once again, this fraction can easily and quickly be checked using your calculator.

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ROUNDING OFF DECIMALS

There are many examples in Maths, particularly with the use of calculators, when we wish to get an approximate answer. The question will always state the accuracy required by asking you to round off the answer.

For example, all problems relating to money must automatically be rounded to the nearest five cents, because that is the smallest denomination of coins available.

> To round off a number correct to a given place, we round up (i.e. add 1) if the next figure is 5 or higher.
> We round down (i.e. leave it the same) if the next figure is less than 5.

Example 1: Round off 58.1367 to 2 decimal places.

58.13 [6] 7

Look at the third decimal place (6). Since this is higher than 5, we must round up by adding 1 to the number 3.
Therefore 58.1367 = 58.14 to 2 d.p.

Note: Another way of asking the same question: 'Round off to the nearest hundredth'.

Example 2: Round off 90.735 to 1 decimal place.

90.7 [3] 5

Look at the second decimal place (3).
Since this is smaller than 5, it must be rounded down by leaving 7 the same.

Therefore 90.735 = 90.7 correct to 1 d.p.



> Another way of asking the same question: 'Round off to the nearest tenth'.

Example 3: Round off 71.995 to the nearest hundredth. This means the same as rounding off to 2 decimal places!

71.99 [5]

Look at the third decimal place (5). Since this number is 5, we must round up by adding 1 to the previous number 9.
Therefore 71.995 = 72.00 correct to the nearest hundredth.

SAMPLE

ROUNDING OFF USING SIGNIFICANT FIGURES

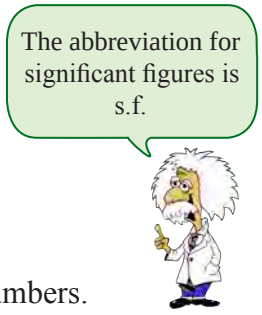
Another frequently used and important method of approximating answers is by rounding off to a stated number of significant (important) figures.

> To approximate to a certain number of significant figures, write down the number which contains the required number of significant figures — and then round up or down according to the next figure.
>
> i.e. Round up if the next figure is ≥ 5
>
> Round down if the next figure is < 5

Students usually find this idea harder than using decimal places because of the problem of zero, which can be significant, or sometimes insignificant, depending on where it lies in the number. The examples below should help to clarify the problems relating to zero.

Examples: Write the following numbers correct to 2 significant figures:

a) 137.25 b) 5.064 c) 0.00342



Solutions: a) The most significant figures are 1 and 3.
Round up, because the next figure (7) is greater than 4.
$\therefore$ 137.25 = 140 to 2 s.f.

b) Zero is significant here because it lies between 2 non-zero numbers.
The 2 most significant figures are 5 and 0.
Round up, because the next figure (6) is greater than 4.
$\therefore$ 5.064 = 5.1 to 2 s.f.

c) The 3 leading zeros are not significant because they don't lie between non-zero numbers.
The 2 most significant figures are 3 and 4.
Round down, because the next figure (2) is less than 5.
$\therefore$ 0.00342 = 0.0034 to 2 s.f.

SAMPLE

LEVEL 1 — FINANCIAL MATHS

EASIER QUESTIONS

Note: Only turn back to page number shown if you have difficulty. For some problems there will be no page reference because it is revision work from Year 8.

Page

Q1. Change the following fractions to decimals: — 2

a) $\frac{18}{40}$ b) $\frac{7}{8}$ c) $\frac{5}{6}$ d) $\frac{11}{12}$

Q2. Change the following decimals to fractions: — 3

a) 0.375 b) $0.\dot{8}$ c) $0.\dot{4}\dot{3}$ d) $4.\dot{3}1\dot{8}$

Q3. Change the following percentages to decimals: — 2, 7

a) $17\frac{1}{4}\%$ b) $48\frac{1}{5}\%$ c) $8\frac{1}{2}\%$ d) 2.1%

Q4. a) Rose scored 21 out of 25 in a Math test. What percentage is this? — Revision from earlier years.

b) Kenneth saves \$37 out of every \$50 that he earns. What percentage of his money does he save?

c) Rick sleeps for 6 hours every night. What percentage of the whole day does he sleep?

d) Elsa grows tomatoes. The birds eat 3 out of every 20 tomatoes. What percentage of tomatoes do the birds eat?



Q5. Randy deposits \$8 000 into a savings account and he earns interest of $2\frac{1}{2}\%$ p.a. — 7, 8

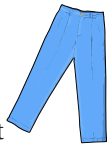
a) What interest does he earn after 1 year?

b) What interest does he earn after 4 years and 9 months?

Q6. Calculate the final prices of the 3 articles below after the discounts have been subtracted: — Revision from earlier years.

a) 
Dress
Normally \$75
20% discount

b) 
Handbag
Normally \$400
38% discount

c) Jeans
Normally \$180
$37\frac{1}{2}\%$ discount


Q7. Julie bought a bicycle for \$800 and sold it 2 years later for \$640. What is her percentage loss? — 6

Q8. Simplify or reduce the following ratios to their lowest terms: — 10

a) 48 : 36 b) $\frac{3}{4}$: 5 c) 45 : 81 d) $3\frac{1}{4}$: $\frac{5}{8}$

Q9. Alvin and Roxanne share a prize of \$2 760 in the ratio of 5 : 3. How much will each of them receive? — 12

Q10. Rodel and Emily share a box of strawberries in the ratio of 7 : 5. If Emily gets 45 strawberries, how many strawberries does Rodel receive? — 12

Q11. How long will it take me to travel 120 km if I am travelling at the following speeds? — 15

a) 50 km/h b) 75 km/h c) 90 km/h d) 72 km/h

SAMPLE

Note: Only turn back to page number shown if you have difficulty.

Page

Q1. Jessica deposits $12 000 into a savings account for 3 years and earns $954 interest. What is the interest rate per year? 7, 8

Q2. The ratio of Jessica's age to her son's age is 9 : 4. After 6 years the new ratio is 2 : 1. What was Jessica's age when the son was born? 11

Q3. In a fruit bowl the ratio of bananas to apples is 7 : 3. If I add 5 more apples to the bowl, there will be 17 apples. What is the ratio of apples to bananas now? 11

How many bananas are on the table?



Q4. Change the following scales to ratio form: 10

a) 1 cm : 2 m b) 1 mm : 1 m c) 1 cm : 1 km d) 1 cm : 10 km

Q5. The scale on a map is given as 1 : 100 000. Find the actual distance between 2 towns which are 3.4 cm apart on the map. 10

Q6. Find the interest earned on the following amounts: 7, 8

a) $8 000 at $4\frac{1}{2}$% p.a. for 2 years b) $16 000 at $3\frac{1}{4}$% for 5 years

c) $12 600 at $2\frac{2}{3}$% p.a. for 30 months c) $9 000 at 3.6% p.a. for 7 months

Q7. The following items already include the GST component. Calculate what the GST amount is. Revision from earlier years.



This is a tricky question. Finding 10% of the amount is NOT correct!

a)  Electric blender $107.25

b)  Electric Frying pan $75.35

c) Kettle $26.73 

Q8. Judy is working at a dress factory. If she works a normal 40 hour week, then she gets paid at $22.00 per hour. For the next 10 hours she get paid time and a half. And if she works more than 50 hours, she gets paid 'double time'. She also gets taxed 32% of her gross earnings each week to get her net wages. 17, 18

a) Calculate her gross earnings if she worked 43 hours during the week.
b) Calculate her net earnings during the same week.
c) Calculate her net earnings if she works for 54 hours in one week.
d) Calculate how many hours she worked in one particular week, when her gross income was $1 111.00.

Q9. Kim-Sun bought a new car, which costs $28 500, if paid for in cash. He paid a $17\frac{1}{2}$% deposit, and repaid the balance owing over a 4 year period at a flat interest rate of $12\frac{1}{8}$%. 21

a) What was his deposit?
b) How much interest did he pay over the 4 years?
c) What is his monthly instalment?

SAMPLE

Q1. The Taxation Scales set by the Australian Taxation office (ATO) change over time.
The Taxation Scale for the 1990 - 1991 year and the 2018 - 2019 year are given below.

Tax Scale: 1990 – 1991

0 – $5250	Nil
$5 251 – $17 650	20.5 cents for each dollar over $5 250
$17 651 – $20 600	$2 542 plus 24.5 cents for every dollar over $17 650
$20 601 – $20 700	$3 264.75 plus 29.5 cents for every dollar over $20 600
$20 701 – $35 000	$3 294.25 plus 38.5 cents for each dollar over $20 700
$35 000 – $36 000	$8 799.75 plus 42.5 cents for each dollar over $35 000
$36 001 – $50 000	$9 224.75 plus 46.5 cents for each dollar over $36 000
$50 001 and over	$15 734.75 plus 47 cents for each dollar over $50 000

Tax Scale: 2018 – 2019

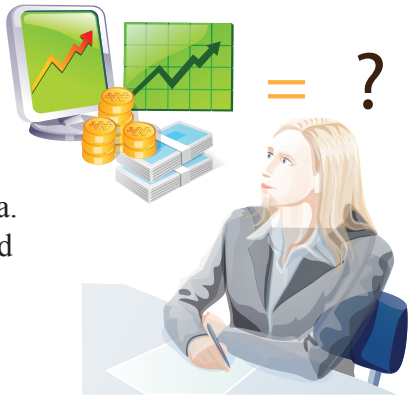
0 – $18200	Nil
$18201 – 37 000	19 cents for each dollar over $18 200
$37 001 – $90 000	$3 572 plus 32.5 cents for each $1 over $37 000
$90 001 – $180 000	$20 797 plus 37 cents for every dollar over $90 000
$180 001 and over	$54 097 plus 45 cents for each dollar over $180 000

a) For an annual salary of $20 000 find:
(i) The tax paid in the 1990 – 1991 year
(ii) The tax paid in the 2018 – 2019 year
(iii) Compare the amount of tax in the 1990 – 1991 year to that paid in the 2018 – 2019 year.

b) (i) If $12 014.75 tax was paid in the 1990 – 1991 tax year what was the annual salary?
(ii) Using the salary found in part (i) find the tax that would be paid in the 2018 – 2019 year
(iii) Compare the amount of tax in the 1990 – 1991 year to that paid in the 2018 – 2019 year.

Q2. Tom, Matt and Neville receive equal shares in an inheritance of $150 000.
a) Matt invests his money into a term deposit which pays 4.8% p.a. simple interest for 3 years.
How much will Matt's investment be worth at the end of the term?

b) Neville invests his money into an account paying 4.5% interest, compounding yearly. If he leaves it in this account for 3 years, will his investment grow to be more or less than Matt's?



c) Tom invests his share of the inheritance for three years in an account which compounds monthly at the rate of 6% pa. Find the difference between his investment and (i) Matt's and (ii) Neville's investment.

SAMPLE

LEVEL 1 – Financial Maths

Q1. a) 0.45 b) 0.875 c) $0.8\dot{3}$ d) $0.9\dot{1}6$

Q2. a) $\frac{3}{8}$ b) $\frac{8}{9}$ c) $\frac{43}{99}$ d) $\frac{4\,314}{999} = 4\frac{318}{999}$

Q3. a) 0.1725 b) 0.482 c) 0.085 d) 0.021

Q4. a) 84% b) 74% c) 25% d) 15%

Q5. a) \$200 per year b) \$950

Q6. a) \$60.00 b) \$284.00 c) \$112.50

Q7. 20% loss

Q8. a) 4 : 3 b) 3 : 20 c) 5 : 9 d) 26 : 5

Q9. Alvin = \$1 725 Roxanne = \$1 035

Q10. Rodel = 63 strawberries

Q11. a) 2 hours 24 min b) 1 hour 36 min c) 1 hour 20 min d) 1 hour 40 min

LEVEL 2 – Financial Maths

Q1. a) Jeremy, \$2.14 b) \$618.44 c) Salary pays \$2 242.50 more d) \$528.94

Q2. a) \$627.76

Q3. a) \$210.56 b) \$5 295.17 c) \$20 199.46 d) \$28 732.78

Q4. a) \$413.40 b) \$1 000 deposit and \$210/month is better by \$131.20

Q5. a) Chocolate ice cream \$4.99/L b) Store 1 is \$22.09 cheaper.

LEVEL 3 – Financial Maths

Q1. Interest rate = 2.65%

Q2. 30 years old

Q3. apples : bananas = 17 : 28 ⇨ There are 28 bananas on the table.

Q4. a) 1 : 200 b) 1 : 1 000 c) 1 : 100 000 d) 1 : 1 000 000

Q5. 3.4 km

Q6. a) \$720.00 b) \$2 600.00 c) \$866.25 d) \$189.00

Q7. We have to divide all the amounts by 11 to find the GST
a) \$9.75 b) \$6.85 c) \$2.43

Q8. a) Gross earnings = 40 × 22 + 3 × 22 × 1.5 = \$979.00
b) Net earnings = \$979.00 less 32% tax = \$979 × 68% = \$665.72
c) (Net earnings = (40 × 22 + 10 × 22 × 1.5 + 4 × 22 × 2) less 32% = \$942.48
d) 40 × \$22 = \$880 ⇨ \$ 1 111 – \$880 = \$231
231 ÷ 33 = 7 ⇨ She worked 47 hours.

Q9. a) Deposit = \$4 987.50 b) \$11 403.56 c) \$727.42

LEVEL 4 – Financial Maths

Q1. a) \$1 074.15 b) \$40.06

Q2. a) \$463 b) 7.5 hours

Q3. a) \$9 938.17 b) 32 hours

Q4. a) Payments: \$20 088.96; Lump sum: \$17 326.71 interest b) 13.4% p.a.

Q5. a) (i) \$1692.75 (ii) \$1 964 (iii) \$2 192.25 b) No, worse off by \$10.88.

SAMPLE