- CLEAR AND CONCISE
- GRADED EXERCISES
- COMPREHENSIVE
- VERY USER FRIENDLY
- SOLUTIONS AT BACK

UNDERSTANDING YEAR 9 MATHS

SAMPLE

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  (Worked solutions to the more difficult exercises.)

SAMPLE

NOTE: The Australian National Curriculum has been split into 3 major strands:

1. Number & Algebra	2. Measurement & Geometry	3. Statistics & Probability
(NA)	(MG)	(SP)

FINANCIAL MATHEMATICS

The 'Australian Curriculum Mathematics' (ACM) references for this sub-strand of 'Number and Algebra' (NA) are given below. This chapter may contain additional extension work which the author feels will be beneficial to the student.

- *Terminating and recurring decimals (ACMNA 184).*
- *Ratios and rates (ACMNA 188).*
- *Percentages and money (ACMNA 189).*
- *Converting rates (ACMNA 208).*
- *Simple interest (ACMNA 211).*
- *Wages, Salaries, Overtime, Income tax (NSW).*

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'The most certain way to succeed is to just try one more time.'

- Thomas Edison (famous inventor)

SAMPLE

TERMINATING AND RECURRING DECIMALS

The decimals shown on the right are called **TERMINATING DECIMALS** because they all have a definite end point.

$$\frac{3}{4} = 0.75$$

$$\frac{12}{20} = 0.6$$

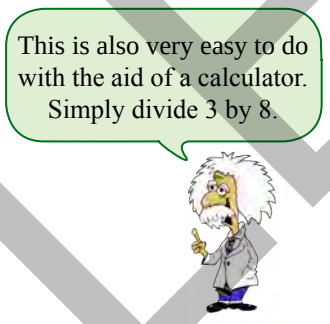
Example 1: Change $\frac{3}{8}$ to a decimal.

Write 3 as a decimal = 3.000

$\frac{3}{8}$ is the same as 3.000 ÷ 8

$$8\overline{)3.^{3}0\,^{6}0\,^{4}0} \quad 0.\ 3\ \ 7\ \ 5$$

Therefore $\frac{3}{8} = 0.375$

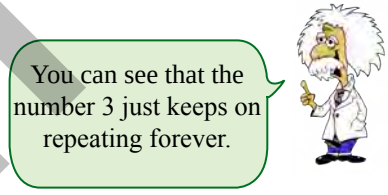


However, when we do the division shown above, we sometimes get decimal answers that never end.

Example 2: Write $\frac{1}{3}$ as a decimal.

$$3\overline{)1.^{1}0\,^{1}0\,^{1}0\,^{1}0} \quad 0.\ 3\ \ 3\ \ 3\ \ 3.....$$

$$\therefore \frac{1}{3} = 0.\dot{3}$$



We place a dot above the 3 to show that this number keeps on repeating infinitely.

When a decimal keeps repeating, it is called a REPEATING or RECURRING DECIMAL.
This is written with a dot on the repeating numbers.

Example 3: Change $\frac{5}{11}$ to a decimal.

Write 5 as a decimal = 5.000

$$11\overline{)5.^{5}0\,^{6}0\,^{5}0\,^{6}0\,^{5}0\,^{6}0\,^{5}0} \quad 0.\ 4\ \ 5\ \ 4\ \ 5\ \ 4\ \ 5....$$

$$\Rightarrow \frac{5}{11} = 0.\dot{4}\dot{5}$$

We place a dot on both the 4 and 5 to show that the pattern 45 repeats itself infinitely.

SAMPLE

Example 4: Change $\frac{254}{999}$ to a decimal.

Try doing this on your calculator and you should obtain the following answer:

254 ÷ 999 = 0.254254254254.....



To show that 254 is a repeating pattern we place a dot over the 2 and also over the 4.

$\therefore$ 0.254 254 254.... = $0.\dot{2}5\dot{4}$

Sometimes you will be required to convert a recurring decimal back into a fraction.

This is done using a clever method involving equations. It is best understood by following the examples below.

Example 5: Convert the following recurring decimals to fractions:

a) $0.\dot{5}$ b) $0.\dot{3}\dot{2}$

Solution: a) Let $x = 0.\dot{5} = 0.55555.....$

$\therefore$ $10x = 5.55555.....$

$10x - x = 5.555.... - 0.5555....$

$\therefore$ $9x = 5$

$\therefore$ $x = \frac{5}{9}$

Subtract the first equation from the second in order to cancel the recurring 5's.



This answer can easily be checked by using your calculator and dividing 5 by 9.

b) Let $x = 0.323232.....$

$\therefore$ $100x = 32.323232.....$

Subtract the first line from the second line

$\therefore$ $100x - x = 32.3232.... - 0.3232.....$

$\therefore$ $99x = 32$

$\therefore$ $x = \frac{32}{99}$

We multiply by 100 because 2 digits are recurring.



Once again, this fraction can easily and quickly be checked using your calculator.

SAMPLE

ROUNDING OFF DECIMALS

There are many examples in Maths, particularly with the use of calculators, when we wish to get an approximate answer. The question will always state the accuracy required by asking you to round off the answer.

For example, all problems relating to money must automatically be rounded to the nearest five cents, because that is the smallest denomination of coins available.

> To round off a number correct to a given place, we round up (i.e. add 1) if the next figure is 5 or higher.
> We round down (i.e. leave it the same) if the next figure is less than 5.

Example 1: Round off 58.1367 to 2 decimal places.

58.13 [6] 7

Look at the third decimal place (6). Since this is higher than 5, we must round up by adding 1 to the number 3.
Therefore 58.1367 = 58.14 to 2 d.p.

Note: Another way of asking the same question: 'Round off to the nearest hundredth'.

Example 2: Round off 90.735 to 1 decimal place.

90.7 [3] 5

Look at the second decimal place (3). Since this is smaller than 5, it must be rounded down by leaving 7 the same.

Therefore 90.735 = 90.7 correct to 1 d.p.



> Another way of asking the same question: 'Round off to the nearest tenth'.

Example 3: Round off 71.995 to the nearest hundredth. This means the same as rounding off to 2 decimal places!

71.99 [5]

Look at the third decimal place (5). Since this number is 5, we must round up by adding 1 to the previous number 9.
Therefore 71.995 = 72.00 correct to the nearest hundredth.

SAMPLE

ROUNDING OFF USING SIGNIFICANT FIGURES

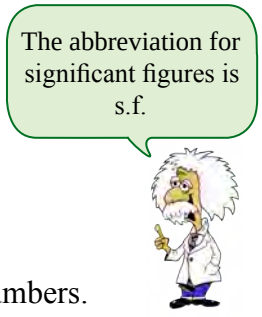
Another frequently used and important method of approximating answers is by rounding off to a stated number of significant (important) figures.

> To approximate to a certain number of significant figures, write down the number which contains the required number of significant figures — and then round up or down according to the next figure.
>
> i.e. Round up if the next figure is ≥ 5
>
> Round down if the next figure is < 5

Students usually find this idea harder than using decimal places because of the problem of zero, which can be significant, or sometimes insignificant, depending on where it lies in the number. The examples below should help to clarify the problems relating to zero.

Examples: Write the following numbers correct to 2 significant figures:

a) 137.25 b) 5.064 c) 0.00342



Solutions: a) The most significant figures are 1 and 3.
Round up, because the next figure (7) is greater than 4.
$\therefore$ 137.25 = 140 to 2 s.f.

b) Zero is significant here because it lies between 2 non-zero numbers.
The 2 most significant figures are 5 and 0.
Round up, because the next figure (6) is greater than 4.
$\therefore$ 5.064 = 5.1 to 2 s.f.

c) The 3 leading zeros are not significant because they don't lie between non-zero numbers.
The 2 most significant figures are 3 and 4.
Round down, because the next figure (2) is less than 5.
$\therefore$ 0.00342 = 0.0034 to 2 s.f.

SAMPLE

LEVEL 1 — FINANCIAL MATHS

EASIER QUESTIONS

Note: Only turn back to page number shown if you have difficulty. For some problems there will be no page reference because it is revision work from Year 8.

Page

Q1. Change the following fractions to decimals: 2

a) $\frac{26}{40}$ b) $\frac{3}{8}$ c) $\frac{1}{6}$ d) $\frac{11}{12}$

Q2. Change the following decimals to fractions: 3

a) 0.625 b) $0.\dot{7}$ c) $0.\dot{2}\dot{3}$ d) $3.8\dot{2}$

Q3. Change the following percentages to decimals: 8

a) 7% b) $48\frac{1}{5}\%$ c) $6\frac{1}{2}\%$ d) $3\frac{1}{4}\%$

Q4. a) Rose scored 19 out of 25 in a Math test. What percentage is this? Revision from earlier years.

b) Kenneth saves \$34 out of every \$50 that he earns. What percentage of his money does he save?

c) Rick sleeps for 6 hours every night. What percentage of the whole day does he sleep?

d) Elsa grows tomatoes. The birds eat 7 out of every 20 tomatoes. What percentage of tomatoes do the birds eat?



Q5. Randy deposits \$12 000 into a savings account and he earns interest of $2\frac{1}{2}\%$ p.a. 9, 10

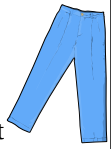
a) What interest does he earn after 1 year?

b) What interest does he earn after 4 years and 9 months?

Q6. Calculate the final prices of the 3 articles below after the discounts have been subtracted: 6

a)  Dress
Normally \$75
22% discount

b)  Handbag
Normally \$400
36% discount

c) Jeans
Normally \$180
$32\frac{1}{2}\%$ discount 

Q7. Julie bought a bicycle for \$800 and sold it 2 years later for \$672. What is her percentage loss? 7

Q8. Simplify or reduce the following ratios to their lowest terms: 12

a) 24 : 36 b) $\frac{2}{3}$: 5 c) 45 : 63 d) $3\frac{1}{4}$: $\frac{7}{8}$

Q9. Alvin and Roxanne share a prize of \$4 480 in the ratio of 5 : 3. How much will each of them receive? 13

Q10. Rodel and Emily share a box of strawberries in the ratio of 7 : 5. If Emily gets 30 strawberries, how many strawberries does Rodel receive? 13

Q11. How long will it take me to travel 120 km if I am travelling at the following speeds? 16

a) 80 km/h b) 75 km/h c) 90 km/h d) 72 km/h

SAMPLE

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AVERAGE QUESTIONS

Note: Only turn back to page number shown if you have difficulty.

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Q1. Round off the following to the specified number of decimal places (d.p.): a) 37.853 (1 d.p.) b) 5.7439 (2 d.p.) c) 49.995 (2 d.p.)	4
Q2. Round off the following to the specified number of significant figures (s. f.): a) 246.31 (2 s.f.) b) 9.5039 (3 s.f.) c) 0.00586 (1 s.f.)	5
Q3. Find the simple interest earned on the following amounts: a) \$8 000 at $3\frac{1}{2}$% p.a. for 2 years b) \$12 000 at $3\frac{1}{4}$% p.a. for 5 years c) \$13 400 at 1.2% p.a. for $3\frac{1}{2}$ years d) \$18 000 at $2\frac{3}{4}$% p.a. for 15 months	9, 10
Q4. Jessica deposits \$8 000 into a savings account for 4 years and earns \$1 120 interest. What is the interest rate per year?	9, 10
Q5. Zac invests \$12 000 in a savings account at 3% p.a. and earns \$1 620. For how long did Zac invest his money?	9, 10
Q6. In a fruit bowl the ratio of bananas to apples was 7 : 3. If I add 3 more apples to the bowl, there will be 15 apples. What is the ratio of apples to bananas now? 	11 - 14
Q7. Change the following scales to ratio form: a) 1 cm : 5 m b) 1 mm : 1 m c) 1 cm : 1 km d) 1 m : 10 km	12
Q8. The scale on a map is given as 1 : 100 000. Find the actual distance between 2 towns which are 4.2 cm apart on the map.	12
Q9. Judy is working at a dress factory. If she works a normal 40 hour week, then she gets paid at \$18.00 per hour. For the next 10 hours she get paid time and a half. And if she works more than 50 hours, she gets paid 'double time'. She also gets taxed 28% of her gross earnings each week to get her net wages. a) Calculate her gross earnings if she worked 43 hours during the week. b) Calculate her net earnings during the same week. c) Calculate her net earnings if she works for 54 hours in one week. d) Calculate how many hours she worked in one particular week, when her gross income was \$1 242.00.	17
Q10. Kim-Sun bought a new car, which costs \$34 000, if paid for in cash. He paid a $17\frac{1}{2}$% deposit, and repaid the balance owing over a 4 year period at a flat interest rate of $9\frac{1}{4}$%. a) What was his deposit? b) How much interest did he pay over the 4 years? c) What is his monthly instalment? 	22

SAMPLE

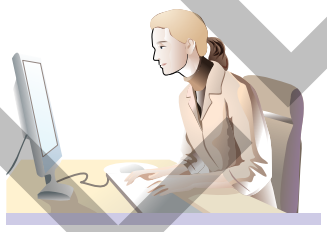
HARDER QUESTIONS

Q1. a) Joydeep receives a normal hourly wage of $23.40 and time-and-a-half for any overtime. On public holidays he receives double-time with a minimum of 4 hours paid. What is his wage for a week in which he works 40 normal hours, 7.5 hours overtime and 4 hours on a public holiday?

b) Karen earns a weekly gross salary of $620. Tax of $148.90 is deducted, 6.5% is paid into a superannuation fund and 8% is paid directly towards her mortgage. Karen aims to save 12% of her remaining income. What amount will she save each week?

Q2. a) Thomas received $547.82 as a holiday loading payment ($17\frac{1}{2}$% on 4 weeks' pay). What is his weekly wage?

b) Kim receives time-and-a-half for any hours worked exceeding the normal 38 hours per week. The normal hourly rate is $16.40. In a particular week Kim receives $795.40. How many hours overtime did he work in that week?

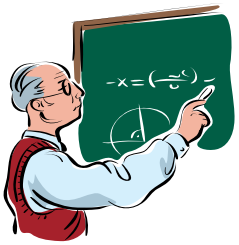


Q3. Becky earns $620 for a 40-hour week. Any overtime is paid at double-time.

a) Using the tax schedule on page 20, what is the tax payable on Becky's normal wage assuming she works for 48 weeks during the year?

b) How many hours overtime can she work in a year and still remain in the same tax category?

Q4. On retirement Victor was offered the option of receiving monthly superannuation payments of $1674.08 or a lump-sum payment of $160 000.



a) What are the respective incomes per month of each option if the lump sum can be invested at 11% p.a.

b) At what interest rate must the lump sum be invested in order for the yearly interest to exceed the yearly superannuation payment?

Q5. Simone is buying a bedroom suite priced at $1 700. She has three payment options:

(i) Pay cash and receive a discount of 8.5%.

(ii) Pay a deposit of $500 and 12 monthly payments of $122.

(iii) Purchase it on finance at 9.25% p.a. payable over 2 years.



a) How much will Simone pay under each option?

b) If Simone pays a deposit of $500 and borrows the remainder at 9.25% p.a. for a 1 year term, will she be better off than if she chose option (ii)?

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LEVEL 1 – Financial Maths

Q1. a) 0.65 b) 0.375 c) $0.1\dot{6}$ d) $0.9\dot{1}\dot{6}$

Q2. a) $\frac{5}{8}$ b) $\frac{7}{9}$ c) $\frac{23}{99}$ d) $3\frac{82}{99}$

Q3. a) 0.07 b) 0.482 c) 0.065 d) 0.0325

Q4. a) 76% b) 68% c) 25% d) 35%

Q5. a) \$300.00 b) \$1 425.00

Q6. a) \$58.50 b) \$256.00 c) \$121.50

Q7. 16% loss

Q8. a) 2 : 3 b) 2 : 15 c) 5 : 7 d) 26 : 7

Q9. Alvin = \$2 800 Roxanne = \$1 680 **Q10.** 42 strawberries

Q11. a) $1\frac{1}{2}$ hours b) 1 hour 36 mins c) 1 hour 20 mins d) 1 hour 40 mins

LEVEL 2 – Financial Maths

Q1. a) 37.9 b) 5.74 c) 50.00

Q2. a) 250 b) 9.50 c) 0.006

Q3. a) \$560.00 b) \$1 950.00 c) \$562.80 d) \$618.75

Q4. Interest = $3\frac{1}{2}\%$ **Q5.** Time = $4\frac{1}{2}$ years

Q6. apples : bananas = 15 : 28 Number of bananas = 28

Q7. a) 1 : 500 b) 1 : 1 000 c) 1 : 100 000 d) 1 : 10 000

Q8. 4.2 km

Q9. a) \$801 00 b) \$576.72 c) \$1 134.00 d) 57 hours

Q10. a) \$ 5 950.00 b) \$10 378.50 c) \$800.59

LEVEL 3 – Financial Maths

Q1. a) $40 \times \$23.40 + 7.5 \times \$23.40 \times 1.5 + 4 \times \$23.40 \times 2 = \$1\,386.45$

b) $\$620.00 - \$148.90 - 6.5\% \times \$620.00 - 8\% \times \$620$

$= \$471.10 - \$40.30 - \$49.60 = \$381.20$

Weekly savings $= 12\% \times \$381.20 = \$45.74$

Q2. a) Let loading payment = $\$x$

$\therefore\ 0.175 \times \$x = \$547.82 \Rightarrow x = \$547.82 \div 0.175 = \$3\,130.40$

$\therefore$ Weekly payment $= \$3\,130.40 \div 4 = \$782.60$

b) Overtime $= (\$795.40 - 38 \times \$16.40) \div (16.40 \times 1.5) = 7$ hours

Q3. a) $48 \times \$620 = \$29\,760$ = yearly salary

Tax $= \$3\,968 + (\$29\,760 - \$17\,651) \times 0.39 = \$8\,690.90$

b) Hourly rate of pay $= \$620 \div 40 = \$15.50$

\$34 000 – \$29760 = \$4 240

overtime hours = \$4 240 ÷ 31 (double-time) = 136.77 (2 d.p.)

She can work a minimum of 136 hours overtime.

Q4. a) $(\$160\,000 \times 11\%) \div 12 = \$1\,466.67$ per month for lump sum payment.

\$1 675.08 per month for monthly superannuation payment.

b) Using $I = P \times R \times N \Rightarrow 20\,088.96 = 160\,000 \times R \times I$

$\therefore\ I = 0.1\,255 \Rightarrow$ Interest = 12.6%

SAMPLE