- CLEAR AND CONCISE
- GRADED EXERCISES
- COMPREHENSIVE
- VERY USER FRIENDLY
- SOLUTIONS AT BACK

UNDERSTANDING YEAR 10 MATHS

SAMPLE

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(Worked solutions to the more difficult exercises.)

Note (i): The Australian National Curriculum has been split into 3 major strands:

1. Number & Algebra	2. Measurement & Geometry	3. Statistics & Probability
(NA)	(MG)	(SP)

Note (ii): The student may think that the last 5 chapters have already been explained in the 'Understanding Year 9 Maths' book by the same authors. However it is very important to note that although these final chapters do certainly revise earlier work, they also explain many new concepts and ideas that have not been included before. In addition, the exercises are more difficult and challenging. Some schools may choose not to teach ALL of the last 5 topics, because they are primarily intended for students who wish to study higher levels of Maths in later years.

SAMPLE

FURTHER FINANCIAL MATHEMATICS

The 'Australian Curriculum Mathematics' (ACM) references for this sub-strand of 'Number and Algebra' (NA) are given below. This chapter may contain additional extension work which the author feels will be beneficial to the student.

- *Earning an income (NSW).*
- *Income tax (NSW).*
- *Simple interest (ACMNA 211).*
- *Compound interest and the formula (ACMNA 229).*
- *Depreciation (NSW).*
- *Time payments (NSW).*

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SAMPLE

Gottfried Wilhelm Leibnitz (1646 – 1716)

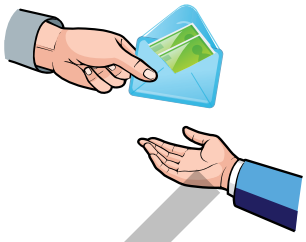


Leibnitz (sometimes referred to as 'The last Universal Genius') was another famous and highly acclaimed German mathematician, logician and natural philosopher who was prolifically active during the 'Age of Reason' or 'Enlightenment' (1685 – 1815). In mathematics his most famous contributions were in the fields of differential and integral calculus which he conceived independently of Isaac Newton, and arguably even before Newton. In fact, most maths works favour the Leibnitz notation as the conventional expression of calulus. He was one of the most ingenious inventors of mechanical calculators and the 'Leibnitz Wheel' was the first mass produced calculator that was ever developed. In addition, he refined the binary counting system which is the foundation of all digital modern day computers and silicon microchips. He also made significant contributions to physics, technology, probability theory, biology, medicine, geology, psychology, linguistics and computer science. If that wasn't enough, he also wrote many works on philosophy, politics, law, ethics, theology, history and philology. His numerous journals were written primarily in Latin, French and German but he also wrote some of his works in English, Dutch and Italian.

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WAYS OF EARNING MONEY

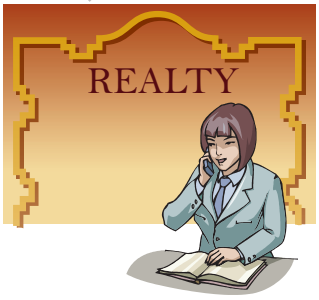
Salary: A person on a 'salary' earns a fixed amount of money per year, usually paid on a fortnightly basis — and these people do not usually receive extra pay for working overtime. A good example is a school teacher.



Wages: Usually paid weekly, and this is calculated on an hourly basis. For many jobs, the Industrial Award allows 'overtime rates' for any additional hours of work beyond the normal working week. These are calculated as follows:

Normal wages = rate of pay per hour × regular hours
Time-and-a-half = rate of pay per hour × overtime hours × 1.5
Double-time = rate of pay per hour × overtime hours × 2

Commission: Many people involved in sales are paid on a commission basis.
Examples include car sales, real estate sales, computer sales, swimming pool sales, kitchen sales, etc. etc.
The commission is expressed as a percentage of the value of the goods sold.



Casual: The person is not employed permanently, but works when required on a fixed rate per hour.
Examples include casual secretaries, casual teachers, gym instructors, etc.



Piece work: This is any type of employment in which a worker is paid a fixed piece rate for each unit produced or action performed regardless of time.
Examples include brick layers, fruit pickers, factory workers.



SAMPLE

ALLOWANCES AND TERMS

HOLIDAY LOADING

People are often paid an extra amount during their annual holiday. This amount is usually calculated as a percentage of the person's usual average wage or salary.

SPECIAL RATES

These are usually paid to wage earners, who obtain an extra rate per hour, for working in dangerous or unpleasant conditions. Examples are: working with poisonous substances, working in confined spaces, working in sewerage, etc.

WEEKEND RATES

People on wages are often paid 'time-and-a-half or 'double-time' for working on Saturdays, Sundays or public holidays.

SUPERANNUATION

This is a lump sum amount of money paid out to a person when they retire. Usually the person and the employer contribute money for many years before retirement to a superannuation fund, and this earns interest on their contributions.

BONUS PAYMENTS

If a company makes a high end of year profit, they sometimes split a percentage of this profit up amongst the employees as a reward for efficient and loyal service.

GROSS PAY

This refers to a person's total weekly wage or salary before any deductions have been made. Net pay refers to the actual 'take home' pay after deductions have been made. These deductions could include income tax, superannuation, union membership, insurance, etc.

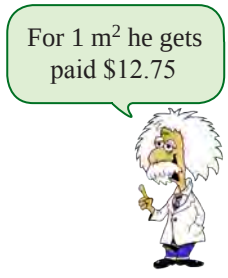
Example 1: Below is a typical time sheet for Mr. F. Jones.
Calculate his wage at the end of the week.

Name	Rate per hour	Regular hours	Overtime		Wage
			Time-and-a-half	Double time	
F. JONES	$17.62	40	7	3	

Normal wages = $17.62 × 40 = $704.80
Time-and-a-half = 17.62 × 7 × 1.5 = $185.01
Double-time = $17.62 × 3 × 2 = $105.72
Total weekly wage = $995.53

Note: Time-and-a-half simply means 'multiply the respective overtime amount by 1.5'.
Double-time simply means 'multiply the respective overtime amount by 2.'

Example 2: A tiler is paid piecework rates at $12.75 per square metre. How much will he earn for tiling a rectangular kitchen floor measuring 3.2 m wide by 4.7 m long?



Area of floor = length × breadth
= 4.7 × 3.2= 15.04 m²
∴ For 15.04 m² he gets paid 15.04 × $12.75 = $191.76

SAMPLE

FURTHER EXAMPLES

Example 1: John Ravenswood works as an architect and earns a salary of $84 728 per year. He is entitled to 4 weeks annual leave per year with an $18\frac{1}{2}\%$ holiday loading. How much would he receive for the 4 week holiday period?



Solution:

One year = 52 weeks
∴ He earns = $84 728 ÷ 52
= $1 629.38 per week
= $6 517.54 per 4 weeks.
Holiday loading = 18.5% × $6 517.54
= $1 205.74
Total holiday pay = $6 517.54 + $1 205.74
= $7 723.28

Example 2: A labourer who normally gets paid $14.31 per hour, also gets a special rate of $2.57 cents/hour when he works in wet conditions.
How much will he earn for 8 hours of work digging trenches on a rainy day?



Solution:

Total amount per hour = $14.31 + $2.57
= $16.88
In 8 hours he earns = 8 × $16.88
= $135.04

Example 3: A real estate salesman charges 5% commission on the first $80 000 of the value of a house, and 2% commission on the remaining balance. How much commission will he earn if he sells a house for $560 000?

Solution:

5% of $80 000 = 0.05 × 80 000 = $4 000
The remaining balance is $560 000 – $80 000 = $480 000
He obtains 2% commission on this remaining balance
2% of $480 000 = 0.02 × 480 000 = $9 600.00
Therefore his total commission is $13 600.



Example 4: Vicki Chan receives a weekly gross salary of $780. If she pays 26% of this in income tax, 5% in superannuation and $41.40 in medical insurance, what is her net weekly pay?

Solution:

Income tax = 26% × $780 = $202.80
Superannuation = 5% × $780 = $39.00
Medical insurance = $41.40
Total deduction = $283.20
Therefore her net weekly pay is $780 – $283.20 = $496.80

SAMPLE

INCOME TAX

The Government has to raise money to pay for its considerable expenses: hospitals, roads, schools, parks, salaries to ministers, etc. One of the ways of raising this money is to tax the yearly income of every individual wage earner — this is called **INCOME TAX**.
It is designed so that people with larger incomes pay more tax than people with smaller incomes. Every person with a certain minimum income must by law complete an income tax return at the end of the financial year (June 30th).



When all the legal deductions have been subtracted from a person's gross earnings, then the 'tax payable' is worked out using a tax table. This table changes from time to time and shouldn't be memorised. However, students must understand how to calculate the tax using a table, because this is a very frequently asked exam question.

Typical Tax Table

Taxable income		Tax on taxable income
From	To	
$1	$5 250	Nil
$5 251	$17 650	Nil plus 32 cents for each $1 in excess of $5 250
$17 651	$34 000	$3 968 plus 39 cents for each $1 in excess of $17 650
$34 001 and over		$10 344.50 plus 47 cents for each $1 in excess of $34 000

As you can see, the table has been divided into 4 major categories of incomes, with each of these on a separate line. Once you have been given a person's taxable income, simply read the information given on the corresponding line, and then do the calculations as stated on that line.

Example: Find the tax payable on the following taxable incomes:
a) $4876 b) $24 700 c) $69 460 d) $15 476

Solutions: a) Nil tax because it lies between $1 and $5 250.

b) This must be worked out using only line 3 on the tax table because it lies in the range $17 651 and $34 000.

i.e. $3968 plus 39 cents for each $1 in excess of $17 650
$= \$3968 + 0.39 \times 7050$
$= \$3968 + \$2749.50$
$= \$6717.50$

Note: $24 700 is $7050 in excess of $17 650.

SAMPLE

LEVEL 1 — FURTHER FINANCIAL MATHS

EASIER QUESTIONS

Note: Only turn back to page number shown if you have difficulty.

Page

Q1. Randy deposits \$12 000 into a savings account and he earns simple interest of $2\frac{1}{2}$% p.a. — 7, 8

a) What interest does he earn after 1 year?

b) What interest does he earn after 6 years?

c) What interest does he earn after 4 years and 9 months?

Q2. Find the simple interest earned on the following amounts: — 7, 8

a) \$8 000 at $3\frac{1}{2}$% p.a. for 4 years

b) \$12 000 at $3\frac{1}{4}$% p.a. for 3 years

c) \$13 400 at 2% p.a. for $3\frac{1}{2}$ years

d) \$18 000 at $2\frac{3}{4}$% p.a. for 18 months

Q3. Jessica deposits \$8 000 into a savings account for 4 years and earns \$960 interest. What is the interest rate per year? — 8

Q4. Zac invests \$12 000 in a savings account at 2% p.a. and earns \$1 800. For how long did Zac invest his money? — 8

Q5. Judy is working at a dress factory. If she works a normal 40 hour week, then she gets paid at \$22.00 per hour. For the next 10 hours she gets paid 'time-and-a-half'. And if she works more than 50 hours, she gets paid 'double-time'. She also gets taxed 26% of her gross earnings each week to get her net wages. — 2

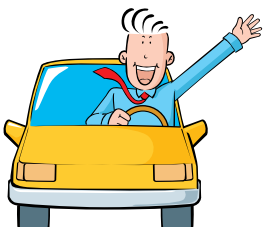
a) Calculate her gross earnings if she worked 43 hours during the week.

b) Calculate her net earnings during the same week.

c) Calculate her net earnings if she works for 54 hours in one week.

d) Calculate how many hours she worked in one particular week, when her gross income was \$1 342.00.

Q6. Kim-Sun bought a new car, which costs \$38 000, if paid for in cash. He paid a $17\frac{1}{2}$% deposit, and repaid the balance owing over a 4 year period at a flat interest rate of $9\frac{1}{4}$%. — 13, 14



a) What was his deposit?

b) How much interest did he pay over the 4 years?

c) What is his monthly instalment?

Q7. A savings account on March 1st is started with \$60 000. The account pays 3.2% interest per annum (on the minimum monthly balance) at the end of the month and \$4 000 is deposited on the last day of each month into the account. Complete the table to find the balance in the account on June 30th. — 9

Date	Balance	Contribution	Interest for the Month
March 1st	\$60 000		
March 31st			
April 1st			
April 30th			
May 1st			
May 30th			
June 1st			
June 30th			

SAMPLE

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LEVEL 2 — FURTHER FINANCIAL MATHS

AVERAGE QUESTIONS

Note: Only turn back to page number shown if you have difficulty.

Page

Q1. Find the simple interest earned for the following: 7, 8

a) \$15 000 at 3% p.a. for 6 years.

b) \$25 000 at $2\frac{1}{4}$ p.a. for 5 years.

c) \$8 500 at 1.7% p.a. for $4\frac{1}{2}$ years.

d) \$42 800 at $1\frac{3}{4}$% p.a. for 2 years 3 months.

Q2. Find the compound interest earned for the following: 9, 10

a) \$8 000 at 5% p.a. for 3 years.

b) \$20 000 at $3\frac{1}{2}$% p.a. for 5 years.

c) \$18 350 at 1.35% p.a. for 4 years.

d) \$9 400 at $2\frac{1}{4}$% p.a. for 8 years.

Q3. The initial cost price of a new car is \$42 500. If it depreciates at 20% p.a. what is its value after 5 years? 11

Q4. Find the compound interest on a principal of \$20 000 borrowed at 6% p.a. for 4 years compounded monthly. 10

Q5. Using the tax table on page 5, calculate the tax payable on the following taxable incomes: 5, 6

a) \$7 825 b) \$24 053 c) \$59 968 d) \$78 124

Q6. a) How much more will Ingrid pay using time payment for a mountain bike worth \$940 if she pays $13\frac{1}{4}$% deposit and \$64.75 per month for 18 months. 13, 14

b) Greg wants to buy a car worth \$7 760. He can either pay a deposit of \$1 000 and then \$215 per month for 36 months, or he can borrow the entire amount at $6\frac{1}{2}$% p.a. simple interest payable over 24 months. Which is the better choice and by how much?



Q7. a) Joydeep receives a normal hourly wage of \$21.40 and time-and-a-half for any overtime. On public holidays he receives double-time with a minimum of 4 hours paid. What is his wage for a week in which he works 40 normal hours, 7.5 hours overtime and 2 hours on a public holiday? 2, 3

b) Karen earns a weekly gross salary of \$820. Tax of \$148.90 is deducted, 6.5% is paid into a superannuation fund and 8% is paid directly towards her mortgage. Karen aims to save 12% of her remaining income. What amount will she save each week?

Q8. a) Thomas received \$602 as a holiday loading payment ($17\frac{1}{2}$% on 4 weeks' pay). What is his weekly wage? 2, 3

b) Kim receives time-and-a-half for any hours worked exceeding the normal 38 hours per week. The normal hourly rate is \$18.40. In a particular week Kim receives \$795.80. How many hours overtime did he work in that week?



Q9. Becky earns \$860 for a 40-hour week. Any overtime is paid at double-time. 2, 3

a) Using the tax schedule on page 20, what is the tax payable on Becky's normal wage assuming she works for 48 weeks during the year?

b) How many hours overtime can she work in a year and still remain in the same tax category?

SAMPLE

Q1. Find the simple interest on a principal of:

a) \$5 893 at 23% p.a. for 3 months.

b) \$16 330 at $9\frac{1}{2}$% p.a. for 32 months.

c) \$27 500 at 4.25% p.a. for 44 days.



Q2. What simple interest rate would allow \$10 110 to grow to \$14 027 in 5 years? [Answer to 2 d.p.]

Q3. Find the compound interest on a principal of:

a) \$6 540 at 6% p.a. for 4.5 years.
For this question, the interest is compounded bi-annually.

b) \$15 430 at $5\frac{1}{4}$% p.a. for 42 months.

Q4. Find the compound interest on a principal of:

a) \$1 040 invested for 20 years at 15% p.a. compounded half yearly.

b) \$30 600 invested for 2 years at 6% compounded quarterly.

c) \$11 920 borrowed at 9% p.a. for 5 years compounded monthly.



Q5. I want to invest \$3 550 for 10 years. I have a choice of investing at a simple interest rate of 15.5% p.a. or a compound rate of 10.25% p.a. Which is the better option and by how much?

Q6. \$7 500 is invested for 3 years with interest compounded biannually. If at the end of the 3 years the investment is worth \$9 767, what is the applied interest rate?

Q7. Find the value after 8 years of:

a) a car costing \$29 000 depreciated at 15% p.a.

b) a printer costing \$569 depreciated at 6.5% p.a.

c) a mobile phone costing \$799 depreciated at $33\frac{1}{3}$% p.a.



Q8. After being depreciated at 7% p.a. for 12 years, a laptop computer is valued at \$413. What was its value 12 years ago?

Q9. Joe and Wendy bought a house costing \$210 000. They paid a 35% deposit and borrowed the remainder at a flat interest rate of 7.75% p.a. payable over 25 years. What is the amount of their total monthly instalment?



Q10. A \$25 000 loan is repaid over 8 years with monthly instalments of \$391. What was the interest rate (p.a.) charged on the loan?



Q11. The tax of Wendy-Li for 1 year was \$21 549.30. Use the tax table shown on page 5 to calculate her gross wage for the year.

SAMPLE

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LEVEL 1 – Further Financial Mathematics

Q1. a) $I = P \times R \times N = \$12\,000 \times 0.025 \times 1 = \$300$

b) $I = P \times R \times N = \$12\,000 \times 0.025 \times 6 = \$1\,800$

c) $I = P \times R \times N = \$12\,000 \times 0.025 \times 4.75 = \$1\,425$

Q2. a) $I = \$8\,000 \times 0.035 \times 4 = \$1\,120$

b) $I = \$12\,000 \times 0.035 \times 3 = \$1\,170$

c) $I = \$13\,400 \times 0.02 \times 3.5 = \$938$

d) $I = \$18\,000 \times 0.275 \times 1.5 = \$742.50$

Q3. $\$960 = \$8\,000 \times R \times 4 \Rightarrow R = \dfrac{960}{4 \times 8\,000} = 0.03 = 3\%$

Q4. $\$1\,800 = \$12\,000 \times 0.02 \times N \Rightarrow N = \dfrac{1\,800}{12\,000 \times 0.02} = 7.5 = 7\frac{1}{2}$ years

Q5. a) $40 \times \$22 + 3 \times \$22 \times 1.5 = \$979$

b) Net earnings $= \$979 \times 74\% = 724.46$

c) $(40 \times \$22 + 10 \times \$22 \times 1.5 + 4 \times \$22 \times 2) \times 74\% = \$1\,025.64$

d) $\$1\,342 - \$880 = \$462$ (1^{st} 40 hours)

$\$462 - \$330 = \$132$ (Next 10 hours)

$\$132 \div \$22 \times 2 = 3$

She worked $40 + 10 + 3 = 53$ hours

Note: Subtracting 26% tax from \$979 is the same as finding 74% of \$979.

Q6. a) Deposit $= \$38\,000 \times 0.175 = \$6\,650$

b) Interest $= \$31\,350 \times 0.0925 \times 4 = \$11\,599.50$

c) Monthly instalment $= (\$31\,350 + \$11\,599.50) \div 48 = \$894.78$

Q7.

Date	Balance	Contribution	Interest for the Month
March 1st	\$60 000		
March 31st	\$64 160	\$4 000	\$160
April 1st	\$64 160		
April 30th	\$68 331.09	\$4 000	\$171.09
May 1st	\$68 331.09		
May 30th	\$72 513.31	\$4 000	\$182.22
June 1st	\$72 513.31		
June 30th	\$76 706.68	\$4 000	\$193.37

LEVEL 2 – Further Financial Mathematics

Q1. a) \$2 700.00 b) \$2 812.50 c) \$650.25 d) \$1 685.25

Q2. a) \$1 261.00 b) \$3 753.73 c) \$1 011.15 d) \$1 831.41

Q3. \$13 926.40 **Q4.** \$5 409.78

Q5. a) \$824.00 b) \$6 465.17 c) \$22 549.46 d) \$31 082.78

Q6. a) \$350.05 b) First option is the better choice by \$28.80

Q7. a) $40 \times \$21.40 + 7.5 \times \$21.40 \times 1.5 + 4 \times \$21.40 \times 2 = \$1\,267.95$

b) $\$820 - \$148.90 - 0.065 \times \$820 - 0.08 \times \$820 = \$552.20$

Each week she will save $12\% \times \$552.20 = \$66.26$

Q8. a) Let x = total wages for 4 weeks.

$\therefore \quad 0.175 \times x = \$602$

$\therefore \quad x = \$602 \div 0.175 = \$3\,440$

$\therefore$ Weekly wage $= \$3\,440 \div 4 = \$860$

b) In 38 hours she gets paid $= 38 \times \$18.4 = \$699.20$

$(\$795.80 - \$699.20) \div (18.4 \times 1.5) = 3.5$

She worked $3\frac{1}{2}$ hours overtime

SAMPLE